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Interaction of Lamb waves with an imperfect joint of plates: reflection, transmission and resonance

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Abstract

The interaction of Lamb waves with an imperfect joint of plates modeled as a linear spring-type interface is analyzed by the elastodynamic finite integration technique (EFIT). For the incidence of the lowest-order symmetric (S0) mode in a low frequency range, the S0-mode transmission coefficient decreases monotonically with the frequency. Furthermore, the imperfect joint exhibits long-time oscillation at certain frequencies depending on the joint stiffnesses when subjected to the S0-mode incidence. These frequencies are in good agreement with the resonance frequencies obtained by the frequency-domain analysis.

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1. Introduction

Elastic waves are widely utilized for detecting and characterizing defects in various structures. Recently, guided waves are attracting much attention in the area of nondestructive evaluation and structural health monitoring due to their ability to propagate long distances. Guided waves in plates, known as Lamb waves, show the complicated behavior at an imperfect joint such as contacting surfaces and adhesive interfaces between solids. This gives rise to difficulty in characterizing such a joint with Lamb waves. In order to understand the interaction of Lamb waves with an imperfect joint of plates, previous studies employed a linear spring-type interface^[1] to model the joint, theoretically elucidating the reflection and transmission characteristics of Lamb waves^[2] at the imperfect joint and the resonance behavior of the joint subjected to the lowest-order symmetric (S0) Lamb mode.^[3]

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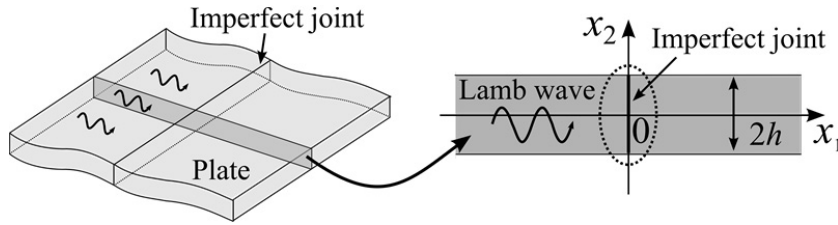


Fig. 1. Elastic plates imperfectly jointed at $x_1 = 0$.

The present study aims to investigate transient responses of an imperfect joint of plates subjected to the Lamb-wave incidence. In particular, partial reflection and transmission of Lamb waves and the resonance behavior of the joint for the S0-mode incidence are analyzed by the elastodynamic finite integration technique (EFIT).^[4] The imperfect joint of plates is modeled as a linear spring-type interface which is characterized by normal and tangential stiffnesses.

2. Numerical analysis

As shown in Fig. 1, two isotropic elastic plates (thickness $2h$) with the same material property are imperfectly jointed at $x_1 = 0$. Under the plane-strain condition in the x_1 - x_2 coordinates, the two-dimensional motions of the plates obey the equation of motion

$$\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} = \rho \frac{\partial^2 u_1}{\partial t^2}, \quad \frac{\partial \sigma_{12}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} = \rho \frac{\partial^2 u_2}{\partial t^2}, \quad (1)$$

where ρ is the mass density, u_1 and u_2 are the displacement components, and σ_{11} , σ_{12} and σ_{22} are the stress components. The plate surfaces at $|x_2| = h$ are assumed to be traction-free, i.e. $\sigma_{22} = 0$ and $\sigma_{12} = 0$. The imperfect joint at $x_1 = 0$ is modeled as a linear spring-type interface^[1]

$$\sigma_{11} = K_N(u_1^+ - u_1^-), \quad \sigma_{12} = K_T(u_2^+ - u_2^-), \quad (2)$$

where K_N and K_T are the normal and tangential stiffnesses of the joint, and a quantity with the superscript $+$ ($-$) represents the value at $x_1 = 0^+$ ($x_1 = 0^-$). In this model, the stresses σ_{11} and σ_{12} are continuous, while the displacement discontinuity is allowed at the joint.

The interaction of Lamb waves with the imperfect joint is analyzed by the elastodynamic finite integration technique (EFIT) proposed by Fellingner *et al.*^[4] The numerical model for the EFIT is shown in Fig. 2. Two aluminum plates (thickness $2h = 1$ [mm] and length of each plate $200h$) are jointed at $x_1 = 0$. The material property of the plates is assumed to be $\rho = 2.7 \times 10^3$ [kg/m³], $c_L = 6.4$ [km/s] and $c_T = 3.17$ [km/s], where c_L and c_T are the velocities of the longitudinal and the shear waves, respectively. Generation and detection of a single Lamb mode via wedges of width $2D = 20h$, longitudinal-wave velocity $c_w = 2.72$ [km/s] and angle θ are modeled. When the incident waveform is denoted by $f(t)$, the normal stress σ_{22} on the plate surface in $|x_1 + 100h| < D/\cos\theta$ is prescribed as $f(t - t_\theta)$, where $t_\theta = (x_1 + 100h)\sin\theta/c_w$ is a time shift depending on the angle θ . The transmitted wave across the imperfect joint

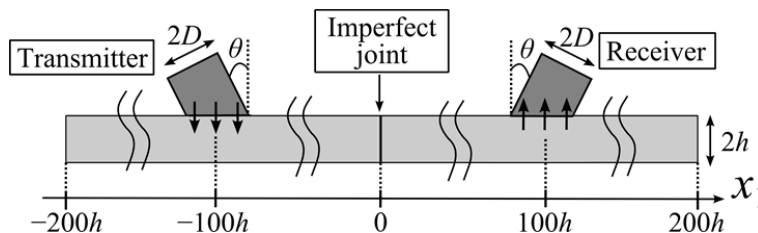


Fig. 2. Numerical model of the plates with an imperfect joint.

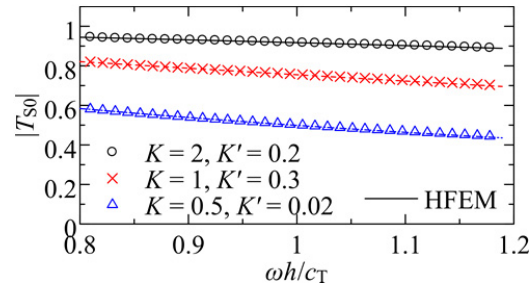


Fig. 3. Variation of the amplitude transmission coefficients of the S0 mode $|T_{S0}|$ with the normalized frequency $\omega h/c_T$ for different normalized joint stiffnesses K and K' .

is calculated as the out-of-plane particle velocity averaged on the plate surface in $|x_1 - 100h| < D/\cos\theta$ after imposing time shifts depending on the position of each calculation point. The angle θ is determined by substituting the phase velocity of the Lamb mode and c_w in Snell's law. The numerical results are summarized by using the normalized frequency $\omega h/c_T$ (ω is the angular frequency), the normalized time tc_T/h , and the normalized joint stiffnesses $K = K_N h/\mu$ and $K' = K_T h/\mu$, where $\mu = \rho c_T^2$ is the shear modulus.

3. Results for the incidence of the lowest-order symmetric (S0) Lamb mode

3.1. Reflection and transmission of the S0 mode in a low frequency range

A Gaussian-modulated tone-burst with the center frequency $\omega h/c_T = 0.991$ is employed as the incident waveform. At this frequency, only the lowest-order Lamb modes can propagate in the plate. The incident angle θ is set as $\theta = 30$ [deg] by using the phase velocity of the S0 mode. The transmitted waveforms of the S0 mode are calculated for various joint stiffnesses and analyzed by the Fourier transform. The amplitude transmission coefficient of the S0 mode is defined as the ratio of the amplitude spectrum of the transmitted S0 mode to that of the incident S0 mode. The amplitude transmission coefficients of the S0 mode $|T_{S0}|$ are shown in Fig. 3, compared to the numerical results of the hybrid finite element method (HFEM) in the frequency domain by Mori *et al.*^[2] The obtained transmission coefficients decrease monotonically with the frequency, showing excellent agreement with the results of HFEM. It is noted that mode conversion between symmetric and antisymmetric modes does not occur since the joint condition is assumed to be symmetric with respect to the neutral plane at $x_2 = 0$.

3.2. Resonance behavior of an imperfect joint of plates

The response of the imperfect joint of plates is investigated for the S0-mode incidence in a higher-frequency range. The incident waveform is a Gaussian-modulated tone-burst with the center frequency $\omega h/c_T = 2.58$. The incident angle θ is set as $\theta = 45$ [deg] by using Snell's law. The transmitted waveforms calculated for two joint stiffnesses are shown in Fig. 4. The waveform in the case of a homogeneous plate (with no joint) is also plotted in Fig. 4 as a reference waveform. The transmitted waves across the imperfect joint show long-time oscillation until $tc_T/h = 950$, while the wave packet passes by $tc_T/h = 750$ in the reference waveform. Le Clezio *et al.*^[5] numerically demonstrated that an analogous phenomenon occurs when a free edge of an elastic plate exhibits the resonance behavior due to the S0-mode incidence. The out-of-plane velocities at the left corner of the imperfect joint $(x_1, x_2) = (0^-, h)$ are plotted in Fig. 5, showing long-time oscillation due to the resonance effect.

The waveforms in Fig. 4 are analyzed by the short-time Fourier transform (STFT) to obtain the time-frequency relations, as shown in Fig. 6. As a result, the frequency components in the long oscillation of the transmitted waves across the joint vary with the normal and the tangential stiffnesses K and K' . By using the HFEM^[3], the resonance frequencies of the imperfect joint are theoretically obtained as $\omega h/c_T = 2.49, 2.66$ when $K = 2$ and $K' = 0.2$, and $\omega h/c_T = 2.54, 2.55$ when $K = 1$ and $K' = 0.3$. The frequencies at which the long-time oscillation is seen in Fig. 6 show good agreement with the resonance frequencies calculated by the analysis of HFEM.

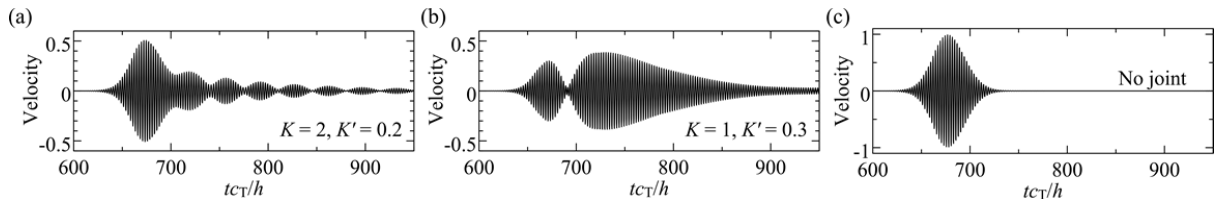


Fig. 4. Waveforms of the transmitted waves for (a) $K = 2, K' = 0.2$ and (b) $K = 1, K' = 0.3$, and (c) the reference waveform calculated for a single plate with no imperfect joint. The vertical axes are normalized by the maximum velocity of the reference waveform.

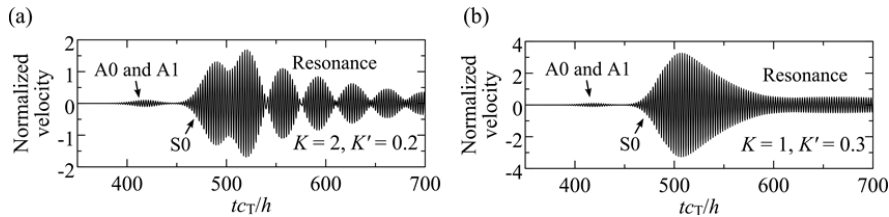


Fig. 5. Normalized out-of-plane velocities at the left corner $(x_1, x_2) = (0^-, h)$ for (a) $K = 2, K' = 0.2$ and (b) $K = 1, K' = 0.3$.

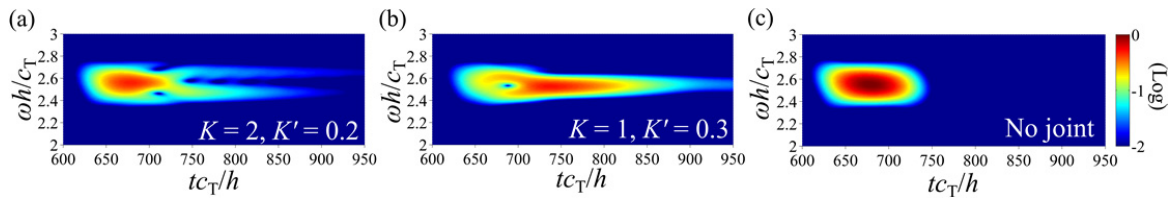


Fig. 6. Time-frequency relations of the amplitude spectra of the transmitted waveforms for (a) $K = 2, K' = 0.2$ and (b) $K = 1, K' = 0.3$, and (c) the reference waveform.

4. Summary

In the present study, the interaction of Lamb waves with an imperfect joint of plates, in particular, (1) the reflection/transmission of the lowest-order symmetric (S0) mode and (2) the resonance behavior of the joint for the S0-mode incidence have been analyzed by the elastodynamic finite integration technique (EFIT). The imperfect joint is modeled as a linear spring-type interface which is characterized by normal and tangential stiffnesses. In a low frequency range, the transmission coefficient of the S0 mode decreases monotonically with the frequency, and show excellent agreement with the previous results obtained in the frequency domain. Furthermore, at certain frequencies depending on the joint stiffnesses, long-time oscillation is seen in the transmitted waveform due to the resonance effect of the imperfect joint in a higher frequency range.

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